Issues for Antiproton Stacking and Cooling

D. McGinnis – June 25, 2003

1 Parameter Goals

To support a luminosity of $3x10^{32}cm^{-2}sec^{-1}$ in the TEVATRON:

- the average antiproton production rate should be at least $40x10^{10}$ antiprotons per hour
- the Antiproton Source complex should be able to stack at this rate for store lengths up to 15 hours
- The antiproton stack size should be at least $625x10^{10}$ antiprotons with a transverse emittance less than 15π mm-mrad (95% normalized) and a longitudinal emittance less than 50 eV-sec.

We will assume that we can collect from the antiproton target 280 x 10^6 8 GeV (kinetic) antiprotons every 2.0 seconds with a transverse emittance of 35π mm-mrad (95% un-normalized) and a momentum spread of 4% confined to bunch lengths of about 1 – 1.5 ns (95% width). We will design for a 95% transfer efficiency from the Debuncher to the Accumulator, a 95% transfer efficiency from the Accumulator to the Recycler, and that the average stacking rate is 90% of the peak stacking rate.

2 Antiproton Stacking Process

2.1 Debuncher

2.1.1 Debuncher Bunch Rotation

After the antiprotons are created at the target, focused by the lithium lens, and transported to the end of the AP2 transfer line they are injected into the Debuncher. The momentum acceptance of the Debuncher is about 4% (350 MeV). The large momentum spread and the short bunches of the antiproton beam are exchanged with a RF bunch rotation[1]. After the bunch rotation, the coasting beam has a momentum spread of about 0.3-0.4%.

2.1.2 Debuncher Stochastic Cooling

The bunch rotation followed by adiabatic debunching takes less than 100 mS.[1] The antiproton beam does not need to leave the Debuncher until the Main Injector is finished accelerating another batch of protons to the antiproton target. This remaining time is used to stochastic cool the beam in all three planes.

Because of the low beam current, the cooling rate of these systems is limited by the amount of power available to the kickers. To increase the gain of these systems, the pickup arrays and the front-end amplifiers are cryogenically cooled. The initial plans for Run II was to change the cryogenic cooling from liquid nitrogen to liquid helium and

[1] TEVATRON I Design Report, Page 4-7, 1984

have plunging 2-4 GHz stripline pickup and kicker arrays. However, in anticipation of the need for higher luminosities later in Run II, the design system bandwidth was changed from 2-4 GHz to 4-8 GHz.[2] Because of the large apertures required in the Debuncher, the stripline array design was abandoned in favor of slow-wave structures.[3] Stripline arrays do not function properly if the beam pipe can support waveguide modes in the desired frequency band. However, slow-wave structures can support fractional bandwidths on the order of 20%, so the 4-8 GHz band is divided into four sub-bands.[4]

The transverse cooling is required to cool the transverse emittance of the beam from injection emittance of 35π-mm-mrad (un-normalized) to 5π-mm-mrad in 1.9 seconds. The momentum cooling system cools the energy spread of the beam after bunch rotation from 35 MeV to 6 MeV in 1.9 seconds.

2.2 Accumulator Momentum Stacking

Just prior to when a new pulse of 120 GeV Main Injector protons is directed to the antiproton production target, the antiproton beam in the Debuncher is extracted and injected into the Accumulator. The Accumulator has a relatively large momentum aperture of 2.5% and the momentum aperture is divided into three regions as shown in Figure 2.2-1. These regions are the injection/extraction orbit, the stacking orbit, and the core orbit. The stacking and core orbits do not see the magnetic field of the injection/extraction kickers. The injected beam is then bunched with a 53 MHz (h=84) RF system and is decelerated about 65 MeV to the high-energy edge of the stacking orbit. The beam is then adiabatically debunched and the Stacktail momentum cooling system is gated on. (It is gated off again just prior to the arrival of the next beam pulse on the high-energy edge of the stacking orbit.)

Comparing designs with different transfer intervals of the Accumulator to Recycler transfer, energy apertures, microwave power levels, and pickup designs, a 2-6 GHz Stacktail system augmented with a 4-8 GHz core momentum cooling system seems to be the best design choice. The job of the 2-6 GHz Stacktail momentum cooling system is to compress about 900 pulses (30 minutes of stacking) of 6 MeV (9.6 eV-Sec.) wide beam from the Debuncher into a single core at the low-energy edge of the Accumulator with a width of about 6.3 MeV (10eV-Sec.). The energy slope of the Stacktail system is 8 MeV and the energy aperture (including the injected pulse and the core) is 58 MeV. The width of the core cooling system is 9.6 MeV with an energy slope of 5 MeV. The amount of power needed to push 45×10^{10} antiprotons/hour though the system is about 350 W for 10 kΩ of kicker impedance.

If the transverse 4-8 GHz core cooling systems are run at about 1.5x the optimum core gain, the transverse emittances will cool through the Stacktail and the core from an initial emittance of 5π-mm-mrad to a final core emittance of about 0.3π-mm-mrad. Assuming that the effective bandwidth of the 4-8 GHz core transverse cooling system is 1.75 GHz instead of the nominal 3.5 GHz, the final core emittance will be about 0.6π-mm-mrad which leaves considerable margin for obtaining the goal of 1.0π-mm-mrad.

---

[2] Pbar Note 573 - Debuncher Stochastic Cooling for Run II and Beyond, J. Marriner, 1998
[3] Pbar Note 626 - Slotted Waveguide Slow Wave Stochastic Cooling Arrays, D. McGinnis, 1999
[4] Pbar Note 625 - The 4-8 GHz Stochastic Cooling Upgrade for the Fermilab Debuncher, D. McGinnis, 1999

When 22.5x10^{10} antiprotons have filled a phase space of 10eV-Sec (6.3MeV width) in the core after about 30 minutes of stacking, the injection process from the Debuncher will be halted and the Stacktail System will be turned off. An h=4 RF system will bunch the core beam in four 2.5 eV-Sec full buckets. These full buckets will be accelerated through the Stacktail region. The portion of the Stacktail that is not captured in the RF buckets will be phase-displaced lower in energy. Once the full buckets clear the Stacktail region, the bucket area will be increased and the buckets will be accelerated to the extraction orbit where the extraction kickers will fire and send the beam into the Accumulator to Recycler transfer line. The empty RF buckets will be reduced in bucket area to 2.5 eV-Sec. and swept through the Stacktail region towards the core. The beam in the Stacktail will then be phase-displaced in the reverse direction higher in energy to its original location. The RF buckets will be turned off and stacking should resume.

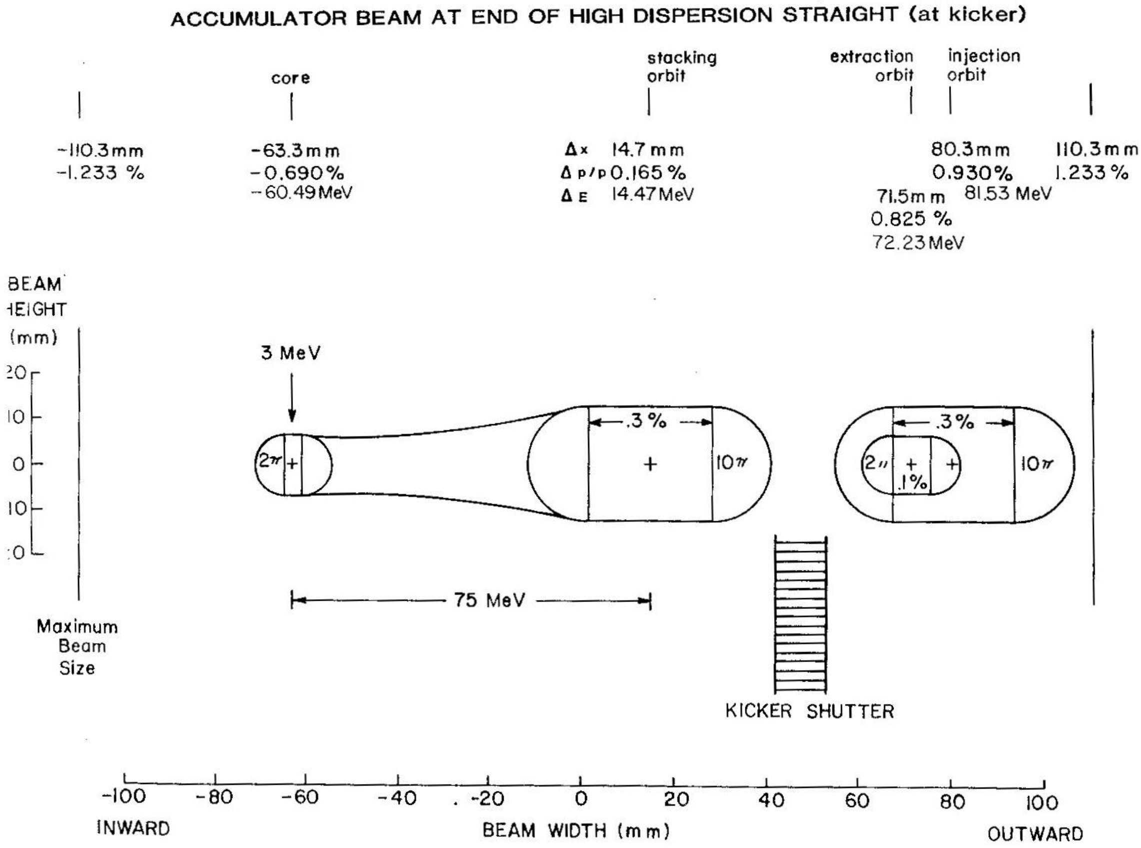

Figure 2.2-1 Accumulator Orbits

2.3 Recycler Cooling

Every thirty minutes, about 22x10^{10} antiprotons in a 10eV-Sec, 1.0π-mm-mrad, phase-space will be transferred from the Accumulator to the Recycler. To keep the interruption to stacking to a minimum, the transfers will be completely automated with the initiation of a timeline event.[5] An emittance dilution of 50% in all three planes and a transfer efficiency of 95% are assumed on each transfer. Since electron cooling is less

[5] Rapid Transfers from the Accumulator to the Recycler, Run II Upgrades, E. Harms, 2003

effective for large transverse emittances, the freshly injected batch of antiprotons from the Accumulator will be stochastically pre-cooled from 1.5π-mm-mrad to 0.3π-mm-mrad. To stochastically pre-cool the injected batch, the injected batch will be kept separate from the main stack with barrier buckets. The 2-4 GHz transverse stochastic cooling systems will have gain gating in which the gain of the cooling systems will be large while the low density injected batch is passing through the cooling electrodes and the gain will be small while the main stack is passing through the cooling electrodes. Just prior to the next transfer from the Accumulator, the injected batch will be merged into the stack with barrier bucket manipulation. Most of the cooling of the stack in all three planes is done with electron cooling while weak stochastic cooling is kept on for high betatron amplitude particles. The electron cooling rate will be about 10 minutes for 500 mA of electron current flowing through a 20 meter cooling section with an electron temperature of 220 μrad.

3 Issues

3.1 Debuncher Issues

3.1.1 Debuncher Bunch Rotation

The final momentum spread of the coasting beam as a function of initial bunch length is shown in Figure 3.1-1.[6] The RF system of the Debuncher can produce 5 MV per turn, which produces a bucket height just large enough to capture the 4% momentum spread of the beam. The non-zero intercept of the curve in Figure 3.1-1 is due to the slower rate of rotation for the particles near the edge of the RF bucket.

[6] Plans for TEVATRON Run IIB. Page 108, 2001

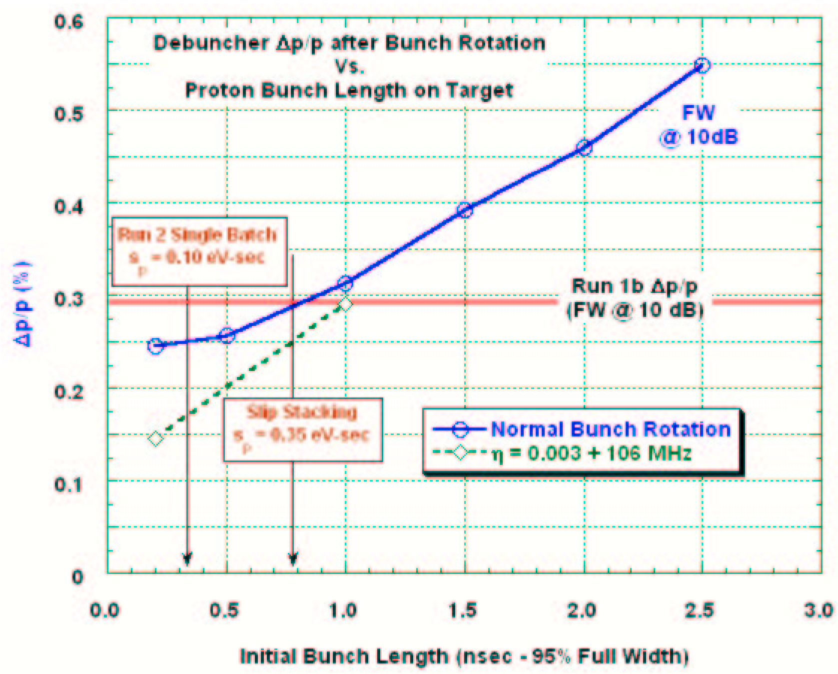

Figure 3.1-1 $\bar{p}$ momentum spread (Δp/p) versus the bunch length of protons on the $\bar{p}$ production target.

Since the calculated longitudinal cooling rate of the Debuncher momentum cooling system is much faster than the Main Injector cycle time, the only constraint on the final momentum spread after Debuncher bunch rotation is that the momentum spread is inside the "aperture" of the momentum cooling system. The cooling aperture is defined by the frequency spread of the Schottky bands at the maximum frequency of the cooling system. For a filter-type momentum cooling system it is commonly regarded that this spread at the maximum frequency of the cooling system should not exceed about one third of the revolution frequency. The momentum aperture of the cooling system is then defined as:

$$\frac{\Delta pc}{pc} = \frac{x}{\eta}\frac{f_o}{f_{max}}$$

(3.1.1)

where x is the spread of a Schottky band in units of revolution frequency (f_o) at the maximum frequency (f_{max}) of the cooling system. Using a maximum frequency of the cooling system as 8.2 GHz, the cooling aperture is about 0.4% for x=1/3. Using the curve shown in Figure 3.1-1, the bunch length on the antiproton production target should not exceed 1.5 nS. There is also a requirement on the phase error of the bunches with respect to the Debuncher RF. Any phase error will cause the bunches to rotate to a different energy than the central energy. It is assumed that any overall phase error will be removed by "tuning" and what is left is any bunch-to-bunch phase variation. The rms value of this phase variation can be added in quadrature to form an effective bunch length:

$$\tau_{95_{eff}} \approx 4\sqrt{\sigma^2_{bunch} + \sigma^2_{phase}}$$

(3.1.2)

3.1.2 Debuncher Transverse Stochastic Cooling

The calculated transverse cooling rate for the four-band 4-8 GHz system with a flux of $400x10^6$ antiprotons per pulse is shown in Figure 3.1-2.[2] The cooling rate time for the four-band system is about 1 second. The initial emittance is 25π-mm-mrad (unnormalized). Note that the graph plots the average emittance. The initial distribution is uniform. The action of the stochastic cooling system will make the distribution more gaussian. The 95% emittance of a guassian is about 3 times the average emittance. The transverse acceptance of the Accumulator is about 8.5π-mm-mrad (un-normalized). However, a conservative design acceptance of 5π mm-mrad is generally used. The amount of beam that is cooled into a 5π-mm-mrad (un-normalized) aperture as a function of cooling time is shown in Figure 3.1-3. At 2 seconds for the four-band system, about 95% of the beam is inside the 5π-mm-mrad acceptance.

However, these calculations were made for $400x10^6$ particles with a starting emittance of 25π-mm-mrad. For a system in which the signal to noise is much greater than one and is power limited, the system gain is proportional to the product of the number of particles and the emittance. (The signal to noise of the four-band system for $100x10^6$ particles and a 25π-mm-mrad emitttance is about one. For $280x10^6$ particles in a 35π-mm-mrad emittance, the signal to noise should be a little less than four.) Using the intensity x emittance scaling argument, the cooling time for $280x10^6$ particles and a 35π-mm-mrad initial emittance should also be near one second. Because of the larger initial emittance of 35π-mm-mrad, the 95% emittance after two seconds of cooling would be greater than 6π-mm-mrad and the transfer efficiency into the Accumulator would be less than 85%. To bring the transfer efficiency up to 95%, the power handling capability of the cooling systems would have to be raised by 25%.

Another option would be to employ gain leveling. As the beam cools for a fixed gain, the power into the kickers will drop. With gain leveling, as the emittance drops the gain could be increased. If the gain was increased linearly with time so that at the end of 2 seconds the gain has increased by 25%, the beam emittance would be under 5π-mm-mrad. The power at the end of 2 seconds with gain leveling would be about 270W compared to the value of 225W without gain leveling.

The above discussion assumes that the beam is centered in the pickups and there is no common mode longitudinal signal detected. If the beam is off center through the pickups, then the ratio of betatron signal power to longitudinal signal power is:

$$\frac{P_\beta}{P_L} = \frac{\beta_{pu}\varepsilon_{95}}{12d^2} \qquad (3.1.3)$$

where β_{pu} is the beta function at the pickup (about 8 meters), ε_{95} is the un-normalized 95% emittance, and d is the amount that the beam is off-center through the pickup. For a 35π-mm-mrad emittance, a tolerance of 1.5mm will give 10 times more betatron power than longitudinal power. Another source of longitudinal signal is phase imbalance between the two sides of the pickups. In this case the ratio of betatron signal power to longitudinal signal power is:

$$\frac{P_\beta}{P_L} = \frac{\varepsilon_{95}}{12A\sin^2(\theta)}$$

(3.1.4)

where A is the acceptance of the pickup and θ is the phase error between the sides of the pickup. For a 35π-mm-mrad emittance and a 40π-mm-mrad acceptance, if the phase error is 15 degrees or the delay imbalance is 7pS at 6 GHz the longitudinal power will equal the betatron power.

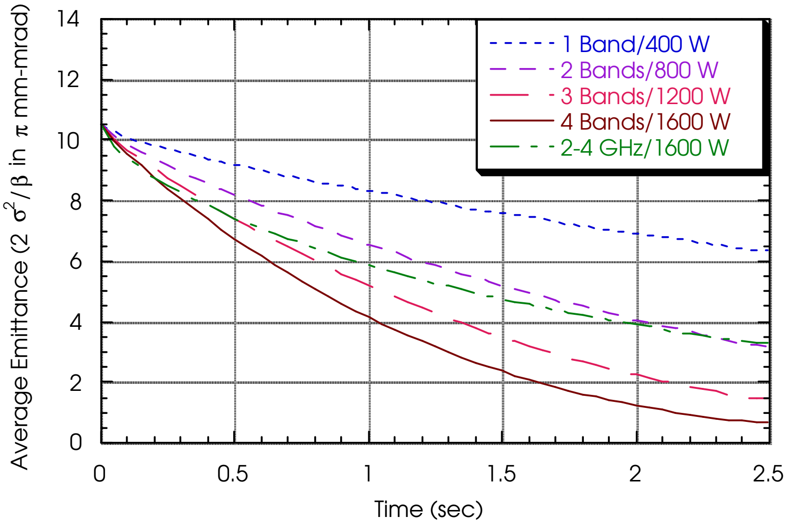

Figure 3.1-2 Horizontal emittance (un-normalized) versus time for 400x10⁶ antiprotons.

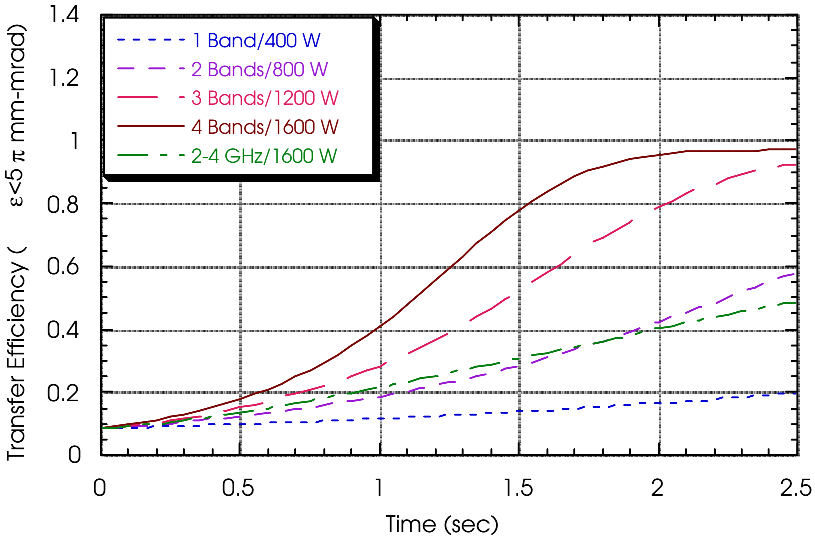

Figure 3.1-3 Transfer efficiency versus cooling time for 400x10⁶ antiprotons. The transfer efficiencies are obtained from the square of the fraction of the beam with an emittance less than 5π-mm-mrad (un-normalized)

3.1.3 Debuncher Momentum Stochastic Cooling

The amount of gain needed in the Accumulator Stacktail system is proportional to the momentum spread of the beam extracted from the Debuncher. A simple model of Debuncher momentum cooling has been constructed.[6] This model does not solve for the time evolution of the momentum distribution directly but determines the cooling rate of the system by taking the second moment of each term in the Fokker–Plank equation. The calculated cooling time for the four-band 4-8 GHz system using 4800W of available kicker power for 280×10^6 particles with an initial momentum spread of 0.4% is 0.2

seconds. The calculated signal to noise is very high, so the final momentum spread at the end of a two second cycle time is calculated to be very small.

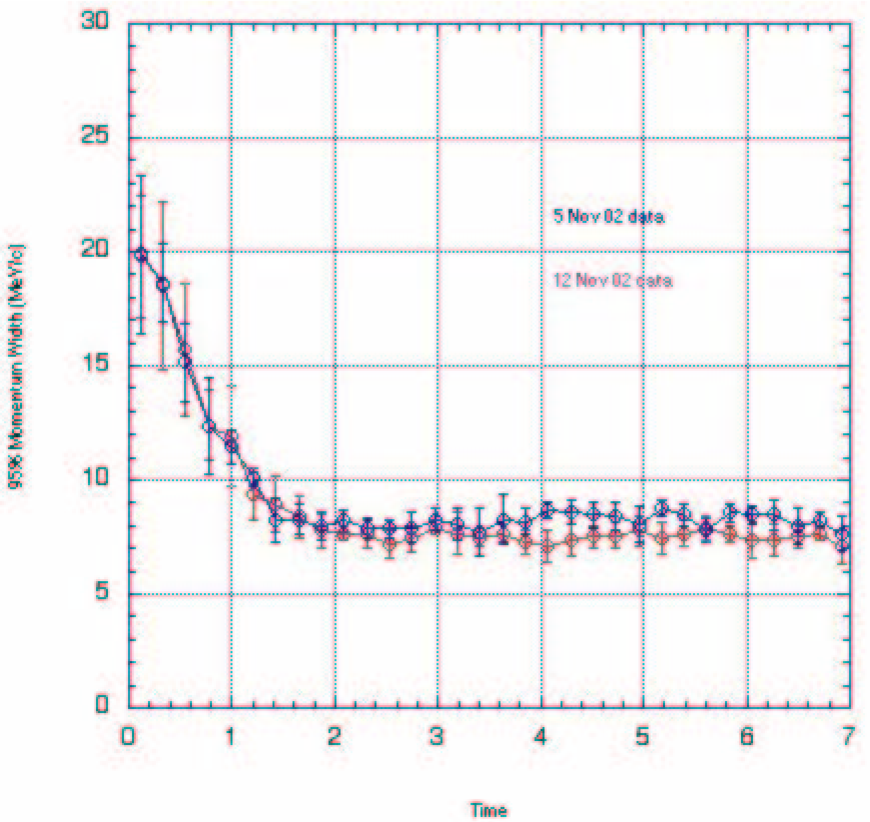

Figure 3.1-4 Debuncher Momentum spread as a function of cooling time

Because the dispersion function in the Debuncher is small (on the order of 2 m), the cooling system uses notch filters to form the momentum error signal. However, this model assumes that there is no dispersion in the notch filters and that the final cooling energy of every one of the four cooling bands is the same. Recent measurements in the Debuncher[7] (shown in Figure 3.1-4) have shown that the asymptotic energy spread of the Debuncher cooling system is about 8 MeV. It is presently thought that band-to-band misalignment of the notch filters, notch filter dispersion, and notch filter depth is the present cause of the large asymptotic energy spread.[8] The energy error caused by notch filter misalignment is given as:

$$\frac{\Delta pc}{pc} = \frac{1}{\eta} \Delta T_{notch} f_o \tag{3.1.5}$$

where ΔT_{notch} is the delay error of the long leg of the notch filter from the desired value of $1/f_o$. A delay error of 1 pS will give rise to an energy error of 1 MeV. The energy spread due to dispersion in the notch filter is given as:

[7] Pbar Note 673 - Debuncher Momentum Cooling Characterization, P. Derwent, 2002
[8] Pbar Note 672 – Performance of the Debuncher Momentum Cooling Notch Filters, R. Pasquinelli, 2002

$$\frac{\Delta pc}{pc} = \frac{1}{\eta}\frac{1}{N}\sum_{n}\frac{\left|\theta_e(nf_o) - \langle\theta_e\rangle\right|}{2\pi n}$$

(3.1.6)

where the sum is over all the Schottky bands in the bandwidth of the cooling system and θ_e is the phase error of the long leg of the notch filter with respect to the short leg of the filter after the delay of $1/f_o$ has been taken out. An average phase error of 2.5 degrees at 6 GHz would give rise to a 1 MeV momentum spread.

3.2 Accumulator Issues

3.2.1 Accumulator Stacktail Momentum Stacking

Because of the complexities of momentum stacking, we will design the Accumulator Stacktail Momentum Cooling system with a safety factor of two. This means that the peak design flux that the Stacktail sytem should handle is $90\text{x}10^{10}$ antiprotons/hour.

The underlying design philosophy of the Stacktail system subject to a constant flux (Φ_o=dN/dt) of antiprotons, is to design a gain profile as a function of energy that maximizes the derivative of the particle density (ψ=dN/dE) with respect to energy ($d\psi/dE$) everywhere. Even though the beam arrives into the Accumulator in lumped pulses, on the time scales of stochastic cooling we can consider the antiproton flux into the Accumulator to be constant. The optimum gain profile is an exponential function of energy.[9,10] For a constant flux, the particle density is inversely proportional to the gain profile V(E).

$$\psi(E) = -\frac{2\Phi_o}{f_o V(E)} = -\frac{2\Phi_o}{f_o V_i} e^{-\frac{E_i - E}{E_d}} = \psi_i e^{-\frac{E_i - E}{E_d}}$$

(3.2.1)

The characteristic energy E_d of the exponential gain profile is given as:

$$\Phi_o = \frac{|\eta| W^2 E_d}{f_o \beta pc \ln\left(f_{max}/f_{min}\right)}$$

(3.2.2)

where W is the bandwidth of the system, f_{max} and f_{min} are the maximum and minimum frequencies of the cooling system. At first glance, Equation (3.2.2) seems to imply that to handle a large antiproton flux one needs to increase the bandwidth, W, and/or E_d. A large value of E_d requires a large physical momentum aperture to hold the distribution. The best way to handle a large antiproton flux is with a large cooling bandwidth.

However, there are other constraints that limit the bandwidth. The flux that the Stacktail system can handle has a logarithmic dependence on the ratio between the maximum and minimum frequencies as shown in Equation (3.2.2). This dependence is due to the poor mixing at low frequencies. Also it is difficult to build microwave systems with large fractional bandwidths. Typically, the largest fractional bandwidth that a single

---

[9] TEVATRON I Design Report, Page 5-14, 1984

[10] An Introduction to the Physics of High Energy Accelerators, D. Edwards, M. Syphers, Page 258, 1993

system can have is an octave. Systems that require a fractional bandwidth greater than an octave usually have to be broken down into several frequency bands.

The constraint on the maximum frequency of the cooling system is determined by system stability. Most of the shaping of the exponential gain profile is done with the pickup design. However, part of the gain profile is shaped with notch filters (especially at the region of high particle density where the gain profile must approach zero). Along with system delay, the necessary presence of notch filters will make the Stacktail system unstable at frequencies where Schottky bands overlap.

A compromise between large bandwidths and system stability is to divide the stacking distribution into two regions as shown in Figure 3.2-1and Figure 3.2-2. The Stacktail region ($E_1<E<E_2$) has lower density, wider momentum aperture, and lower bandwidth than the core region ($E_2<E<E_3$). We will assume that the bandwidth of the core region is 4-8 GHz and is a system that is similar in design to the present operational Core 4-8 GHz Momentum Cooling system.

To stay below frequencies where the Schottky bands overlap, the maximum frequency of the Stacktail system must be below:

$$f_{max\_stack} < \frac{f_o}{|\eta|} \frac{pc}{\Delta E_s + \Delta E_c + \Delta E_{bD}} \tag{3.2.3}$$

where ΔE_{bD} is the width of the beam that is deposited into the Stacktail every cycle and is equal to the final energy spread of the Debuncher. For the present operational 2-4 GHz Stacktail system, the frequency where the Schottky bands overlap is 5.3 GHz.

Decreasing η can increase the maximum Stacktail frequency. Since the maximum flux through the Stacktail system is proportional to the bandwidth squared, the maximum flux will increase. This strategy was followed for the initial Run II upgrades. One of the drawbacks of this strategy is the cooling rate of the transverse core cooling systems will be slower for a fixed bandwidth. Also, the lattice changes required to reduce η have a number of undesirable features. First, the beta functions around the ring had to increase resulting in a smaller aperture.[11] Second the horizontal emittance growth rate due to intra-beam scattering was increased dramatically.[12]

The momentum aperture of the core system (ΔE_c) is determined by the bad-mixing limit. Bad mixing arises due to transit time differences between pickup to kicker of different momentum particles. If we allow a maximum phase error of 30 degrees between a particle with energy E_2 and a particle with energy E_3 at the maximum frequency of the system with a system delay that encompasses one half of the ring, then the energy aperture of the system is:

$$\Delta E_c \le \frac{1}{6\eta} \frac{f_o}{f_{max\_core}} pc \tag{3.2.4}$$

For a maximum frequency of 8 GHz, the width of the core region is limited to less than 9.6 MeV.

---

[11] Pbar Source Upgrades and Commissioning, D. McGinnis, May 2000, http://www-bdnew.fnal.gov/pbar/organizationalchart/mcginnis/Talks/Beam_Seminar_5_23_00/index.htm

[12] Status of the Pbar Source, D. McGinnis, May 2002, http://www-bdnew.fnal.gov/pbar/organizationalchart/mcginnis/Talks/acc_5_13_02.pdf

The particle distribution in the Stacktail region can be written as:

$$\psi_s(E) = \psi_1 e^{\frac{E-E_1}{E_{ds}}}$$

(3.2.5)

The starting distribution density ψ_1 is the number of particles per pulse (Main Injector cycle) divided by the final energy spread (ΔE_{bD}) of the Debuncher. Since the antiproton flux is the number of particles per pulse divided by the cycle time (T_{rep}), the initial distribution is written as:

$$\psi_1 = \frac{\Phi_o T_{rep}}{\Delta E_{bD}}$$

(3.2.6)

The particle distribution in the core is:

$$\psi_c(E) = \psi_s(E_2) e^{\frac{E-E_2}{E_{dc}}}$$

The number of particles transferred to the Recycler is equal to the flux multiplied by the amount of time between transfers (T_{stack}) to the Recycler (assuming that the actual transfer time is negligible). Since these particles must be contained inside the desired longitudinal emittance (A_{bc}), the time between transfers (T_{stack}) is constrained by:

$$\Phi_o T_{stack} = \int_{E_3 - f_o A_{bc}}^{E_3} \psi_c(E) dE$$

$$T_{stack} = T_{rep} \frac{E_{dc}}{\Delta E_{bd}} \left(1 - e^{-f_o A_{bc}/E_{dc}}\right) e^{\frac{\Delta E_s}{E_{ds}} + \frac{\Delta E_c}{E_{dc}}}$$

(3.2.7)

The final parameter of interest is the amount of kicker power required. Since the particle density and the gain distribution is known, the microwave power needed to move the maximum flux through the Stacktail system is given as:

$$P_{\Phi\max} = 2 \frac{f_o}{W} \Phi_o T_{rep} \frac{1}{Z_k} \left(\frac{\Delta E_{bD}/q}{f_o T_{rep}} \right)^2 \frac{E_{ds}}{\Delta E_{bD}} e^{\Delta E_{bD}/E_{ds}}$$

(3.2.8)

where Z_k is the total kicker impedance. The present operational 2-4 GHz Stacktail system has an effective kicker impedance of about 6400Ω. (This formula is correct only for $\Delta E_{bD} << E_{ds}$.) If the amount of flux pushed through the system is less than the maximum flux supported by the exponential slope, the power needed is reduced by:

$$P(\Phi) = P_{\Phi\max} \left(\frac{1 - \sqrt{1 - \frac{\Phi}{\Phi_{max}}}}{\frac{\Phi}{\Phi_{max}}} \right)^2$$

(3.2.9)

Equations (3.2.1) through (3.2.9) can be used to compare the performance of the Stacktail system with different design parameters. We will consider three different designs with Stacktail bandwidths of 2-4 GHz, 2-6 GHz, and 4-8 GHz. Each design will have a core system with a bandwidth of 4-8 GHz. The energy slope E_d for the three different systems is shown in Table 3.2-1. The bandwidth-squared dependence of the energy slope gives a huge advantage of the 2-6 GHz and the 4-8 GHz systems over the 2-4 GHz system. The mixing effect gives a slight advantage of the 4-8 GHz system over the 2-6 GHz system.

Stacktail Bandwidth (GHz)	Core Bandwidth (GHz)	E_{ds} (MeV)	E_{dc} (MeV)	$\Delta E_s+\Delta E_{bd}$ (MeV)	ΔE_c (MeV)	Fraction Unstacked (%)
2-4	4-8	20	5	77.4	9.6	50
2-6	4-8	8	5	48.4	9.6	66
2-6	2-6	8	8	45.2	12.8	55
4-8	4-8	5	5	33.9	9.6	72

Table 3.2-1 Stacktail Design Parameters

Using the same design margin as the present operational 2-4 GHz system, the energy apertures for each of the systems is calculated using Equation (3.2.3) and is listed in Table 3.2-1. The low maximum frequency of the 2-4 GHz system allows it to use more of energy aperture than the other two systems. For the 4-8 GHz system, the injected pulse together with the core will use over 25% of the useable momentum aperture.

Once the energy aperture and the energy slope is known, the maximum number of beam pulses that can be compressed into the available phase space of the core can be calculated using the Equation (3.2.7). The maximum amount of stacking time between Accumulator to Recycler transfers is shown in Figure 3.2-3. The amount of beam unstacked at each transfer for a flux of 45×10^{10} antiprotons per hour is shown in Figure 3.2-4. The power required for each of the systems is shown in Figure 3.2-5.

The 2-4 GHz system requires an extremely short interval of stacking time between Accumulator to Recycler transfers. The 4-8 GHz system can stack as long as 80 minutes before a transfer is needed when the Debuncher energy spread is 4 MeV. The 2-6 GHz system can stack for about 55 minutes with the same Debuncher momentum spread. (A 2-6 GHz Stacktail system without a 4-8 GHz Core system can only stack for 35 minutes given a 4 MeV Debuncher momentum spread.) However, the advantage of the 4-8 GHz system over the 2-6 GHz system quickly diminishes for a larger Debuncher momentum spread. This is because a large Debuncher momentum spread would use up too much of the available energy aperture.

The 4-8 GHz system has the best performance of the three systems considered. For a Debuncher momentum spread of 4 MeV, a Recycler to Accumulator transfer interval of 80 minutes is very attractive. The 2-4 GHz system with a transfer interval of 8 minutes does not seem workable. The main difference between the 2-6 GHz system and the 4-8 GHz system is the poorer mixing factor for the 2-6 GHz system and the smaller energy aperture for the 4-8 GHz system. A transfer interval of 30 minutes with a Debuncher energy spread of 6 MeV for the 2-6 GHz system is not unreasonable. Since the fractional bandwidth of the 2-6 GHz system is larger than an octave, the system would most likely be composed of a 2-4 GHz and a 4-6 GHz band. This extra complication of two bands for the 2-6 GHz system does not seem as near as difficult as building pickup and kicker electrodes that work up to 8 GHz in a very wide over-moded beam pipe for the 4-8 GHz system. Because of the reasonable transfer time, the larger energy aperture, and the simpler pickup electrode design, the 2-6 GHz Stacktail system augemented with a 4-8 GHz core system looks like the best choice.

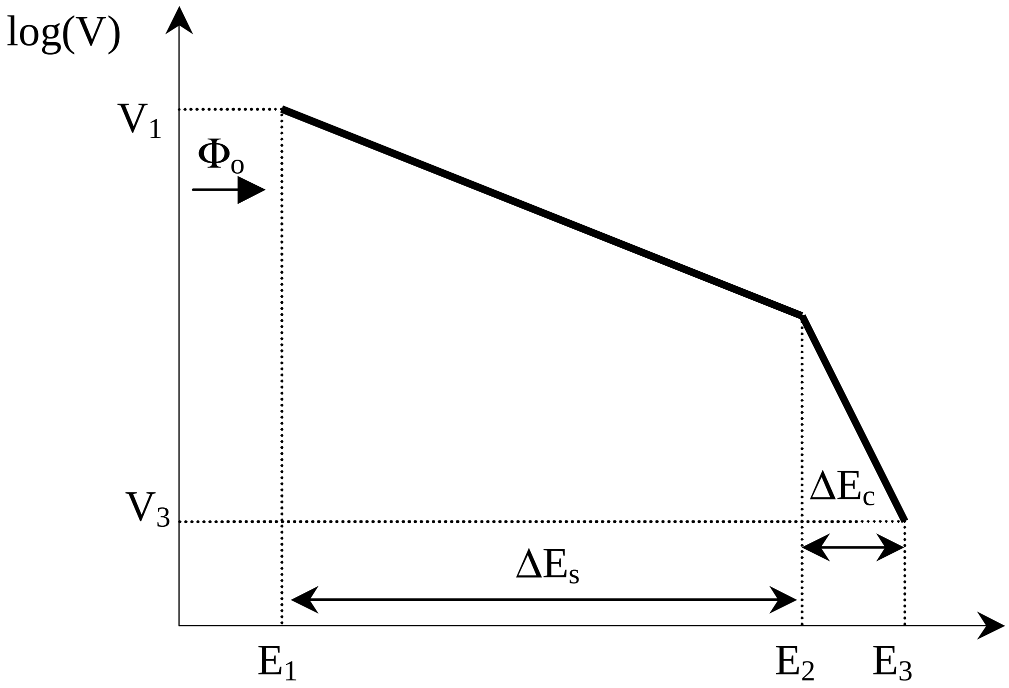

Figure 3.2-1 Stacktail Momentum Cooling System Exponential Gain Profile

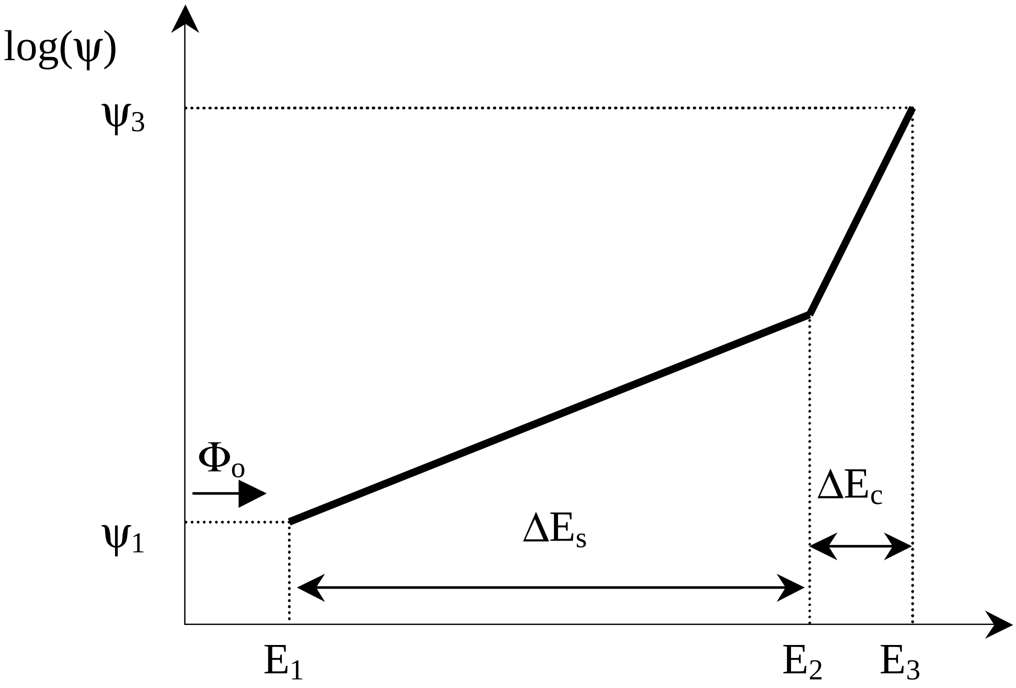

Figure 3.2-2 Particle Density in the Stacktail

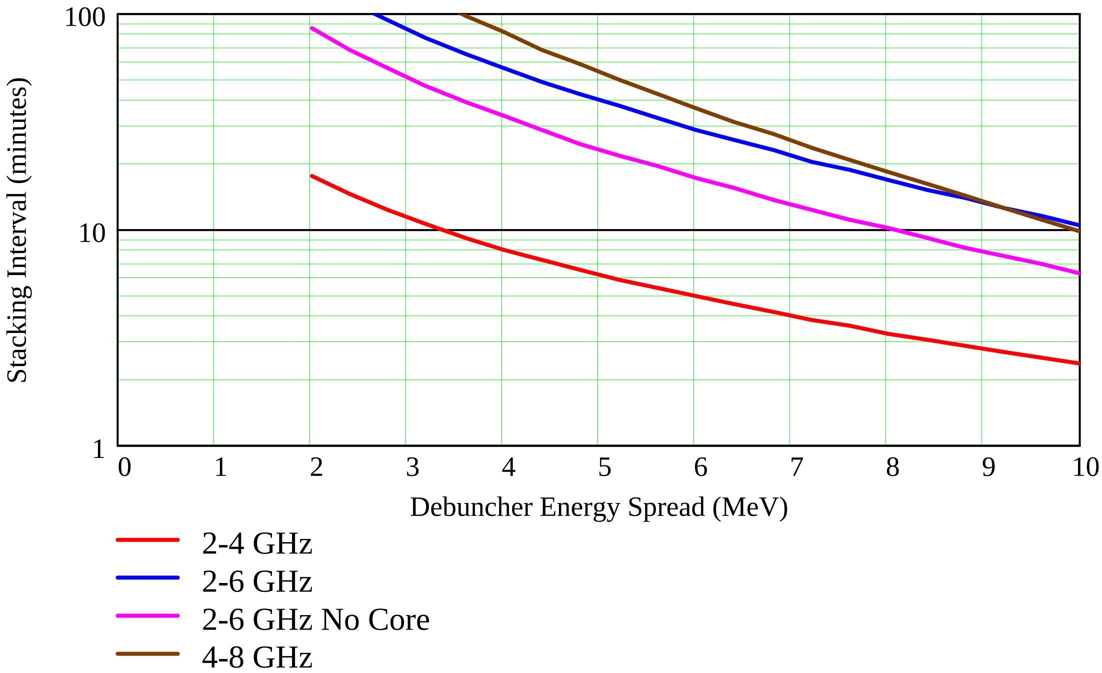

Figure 3.2-3 Stacking interval required to accommodate a flux of $90x10^{10}$ antiprotons per hour for different Accumulator Stacktail Momentum system bandwidths.

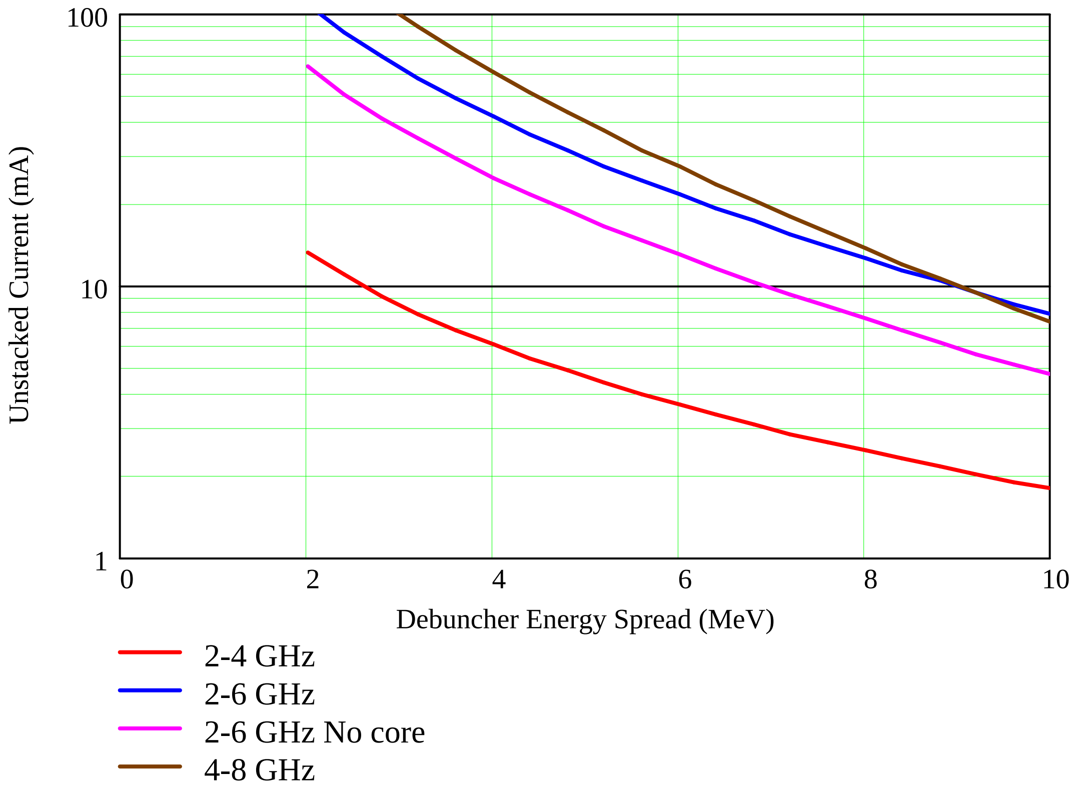

Figure 3.2-4 Amount of beam unstacked for a flux of 45x10^{10} antiprotons per hour

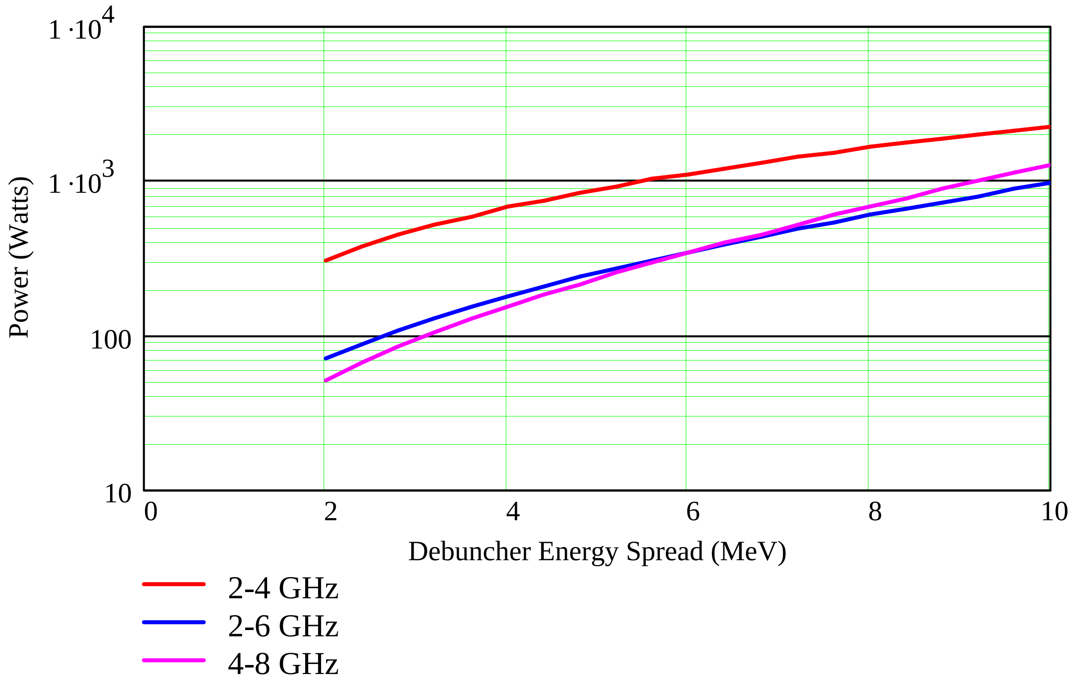

Figure 3.2-5 Microwave power needed for a flux of 45x10^{10} antiprotons per hour for 10kΩ of kicker impedance.

3.2.2 *Accumulator Transverse Cooling*

The transverse emittance of the beam transferred from the Debuncher is designed to be less than 5π-mm-mrad (95%, un-normalized). Electron cooling in the Recycler requires the beam transferred from the Accumulator to be less than 1.5π-mm-mrad. The only transverse cooling that is presently done in the Accumulator is with 4-8 GHz systems designed to cool only the core beam in both planes.[12] Each plane of this system is comprised of three separate sub-bands with bandwidths of about 1.2 GHz centered at 4.8, 6, and 7.2 GHz. The effective bandwidth of the entire system is about 3.5 GHz. The transverse cooling equation is usually written as:

$$\frac{d\varepsilon}{dt} = -\frac{W}{N}\left[2\,\mathrm{Re}\{g\} - |g|^2\left(\frac{1}{n_l}\sum_n \frac{f_o}{\Delta f_n}\right) - |g|^2 U\right]\varepsilon \tag{3.2.10}$$

where the sum is over all the Schottky bands inside the bandwidth W, n_l is the number of Schottky lines, and U is the average noise to average signal.[13] This equation is written in a form that is most useful for understanding cooling of a single core of particles. Since the antiprotons as they move through the Stacktail will see a wide range of particle density, the equation can be rewritten in a more useful form:

$$\frac{d\varepsilon(t,E)}{dt} = -\frac{1}{\tau_c}\left(2\,\mathrm{Re}\{x\} - |x|^2 \frac{M(E)}{M(E_c)}\right)\varepsilon(t,E) + \frac{|x|^2}{\tau_c}\frac{U_o}{M(E_c)} \tag{3.2.11}$$

where for a uniform gain:

$$M(E) = \frac{pc}{\eta}\psi(E)\frac{1}{n_l}\sum_n \frac{1}{n} \tag{3.2.12}$$

τ_c is the optimum cooling time at energy E_c:

$$\frac{1}{\tau_c} = \frac{W}{M(E_c)} \tag{3.2.13}$$

and x is the ratio of the gain to the optimum gain. U_o is a constant of the system and can be determined by measuring the signal to noise S_m for a given emittance ε_m and mixing factor $M(E_m)$:

$$U_o = \varepsilon_m \frac{M(E_m)}{S_m} \tag{3.2.14}$$

For the present 4-8 GHz transverse core-cooling systems, U_o has an approximate value of $240\text{x}10^9\pi$-mm-mrad.

We will assume that the magnitude of the electronic gain of the cooling system does not vary as a function of energy and that the system is phased for only energy E_c. Particles with energy different than E_c will have a phase error at harmonic n between pickup and kicker given as:

---

[13] Stochastic Cooling Theory, CERN-ISR-TH/78-11, F. Sacherer, 1978

$$\Delta\theta_n(E) = 2\pi n \frac{\eta}{3}\frac{E - E_c}{pc}$$

(3.2.15)

where the factor of three is because the distance from pickup to kicker is one third the circumference of the Accumulator. Then cooling term in Equation (3.2.11) is replaced with:

$$2\,\mathrm{Re}\{x\} = 2x_o \frac{1}{n_1}\sum_n \cos(\Delta\theta_n(E))$$

(3.2.16)

To calculate the transverse emittance of a sample of particles as they travel through the Stacktail system, the energy of the sample at a given time must be known. The time t at which the sample of particles is at energy E is given as:

$$t = \frac{1}{\Phi_o}\int_{E_1}^{E} \psi(\xi)d\xi$$

(3.2.17)

The time it takes a sample of particles to traverse the Stacktail and the core is shown in Figure 3.2-6. Because the entire stack is not removed during transfers to the Recycler, the time it takes a sample of particles to travel across the Stacktail system is substantially larger than the transfer interval between the Accumulator to the Recycler.

The final transverse emittance as the particles finish traveling through the Stacktail and the Core momentum systems can be found by integrating Equation (3.2.11). It will be assumed that the transverse core cooling systems will be phased for particles at the edge of the core distribution ($E=E_3$). The relative gain x will be referenced to this energy as well. The final transverse core emittance for a relative gain of one is shown in Figure 3.2-7. The 2-6 GHz Stacktail system with the 4-8 GHz core momentum system has a final emittance of 0.3π-mm-mrad. (The 2-6 GHz system without the 4-8 GHz core system has a substantially lower transverse emittance because of the lower particle density in the core and the larger momentum aperture.)

This calculation was done for optimum cooling at the core densities. However, the particles spend most of the time away from the core. At the lower particle densities away from the core, the heating term due to mixing is not important and the cooling rate would be faster at these locations if the cooling gain was increased. Figure 3.2-8 shows the final transverse core emittance for large relative core cooling gains with a 6 MeV Debuncher momentum spread. A relative core cooling gain of 1.6 would reduce the final transverse core emittance to 0.22π-mm-mrad for the 2-6 GHz Stacktail system with a 4-8 GHz core momentum system.

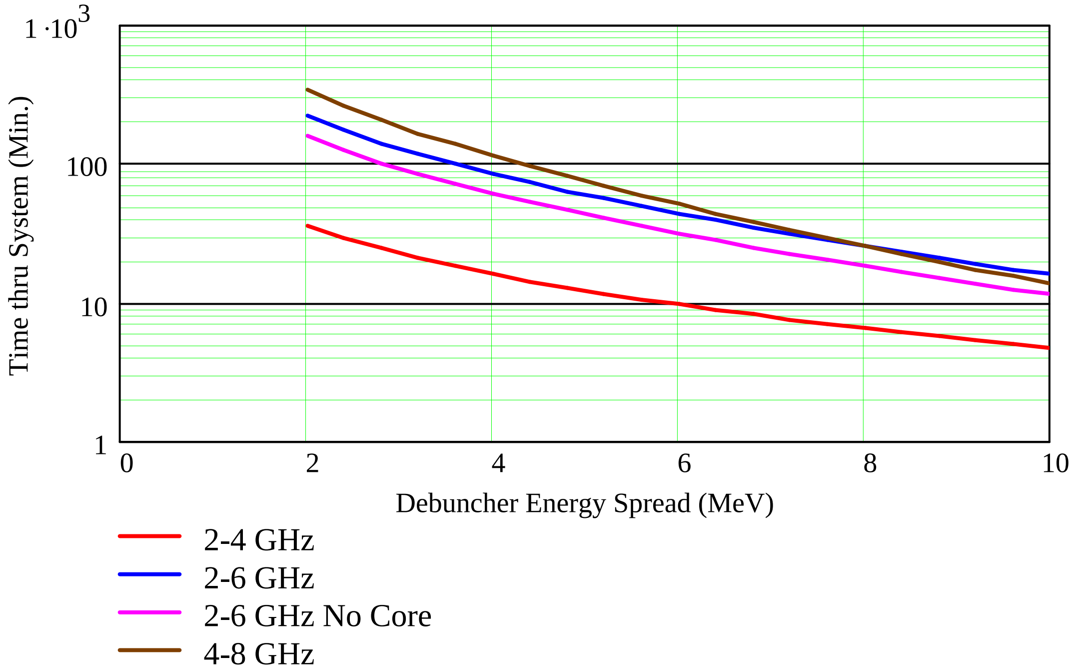

Figure 3.2-6 Time it takes a sample of particles to travel across the Stacktail and Core Momentum cooling systems.

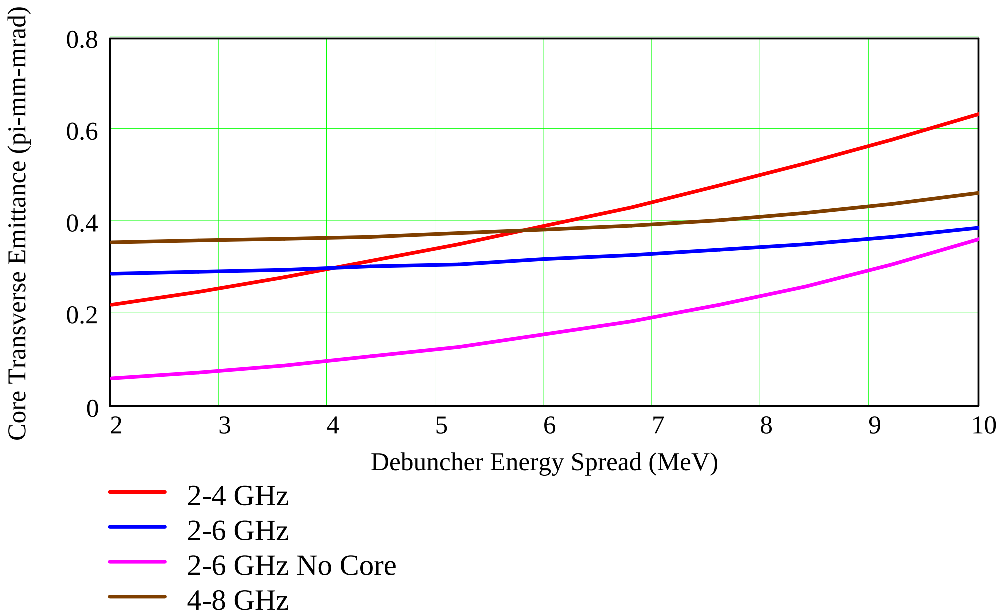

Figure 3.2-7 Final transverse core emittance for a relative gain of one.

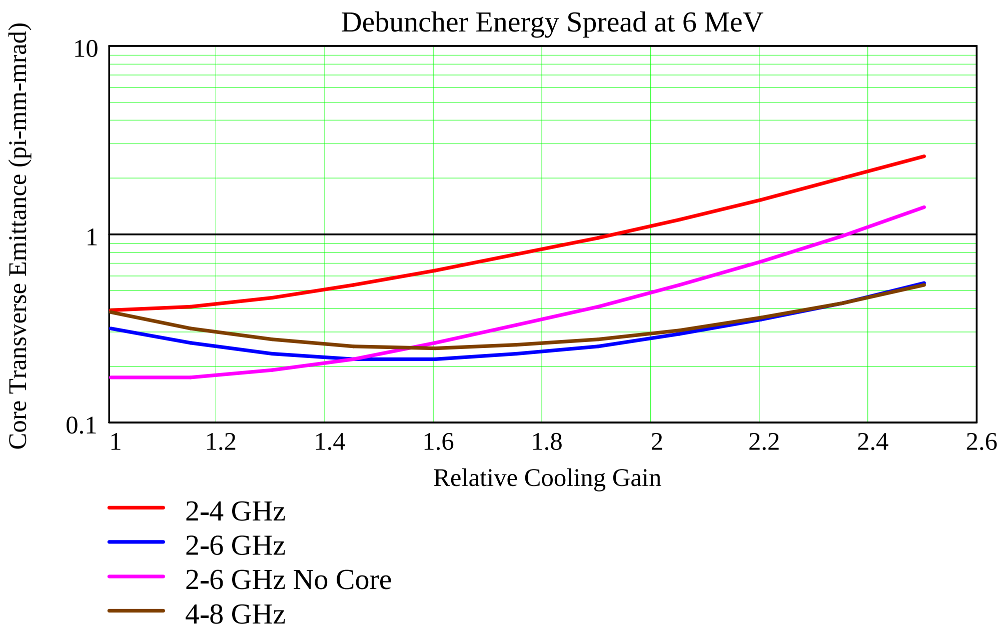

Figure 3.2-8 Final transverse core emittance as a function of relative gain at the core for a Debuncher Energy spread of 6 MeV

3.3 Recycler Issues

3.3.1 Recycler Electron Cooling Considerations

Electron cooling in the Recycler is needed mostly for longitudinal cooling. The instantaneous longitudinal cooling rate (averaged over betatron oscillations) for a given antiproton is proportional to the electron beam density divided by the product of the transverse velocity squared and the longitudinal velocity (in the rest frame of the beam):

$$\lambda_{\parallel} \propto \frac{n_e}{v_{\perp}^2 v_{\parallel}} \qquad (3.3.1)$$

The transverse cooling rate (averaged over betatron oscillations) of a particle is proportional to the electron beam density divided by the transverse velocity of the particle cubed:

$$\lambda_{\perp} \propto \frac{n_e}{v_{\perp}^3} \qquad (3.3.2)$$

where $v_{\perp}, v_{\parallel}$ are amplitudes of betatron and longitudinal velocities. These simplified formulas assume a zero-temperature electron beam and $v_{\perp} > v_{\parallel}$ for antiprotons in the cooling section. They also assume that a particle executes its betatron oscillations inside of the electron beam. In fact, betatron phases where the particle is almost at rest have the highest contribution to the longitudinal cooling rate.

One consequence of these cooling properties is that an antiproton with a betatron amplitude greater than the electron beam radius is practically not cooled longitudinally. Since the transverse electron beam density is uniform with sharp edges and the antiproton beam density is gaussian there is an issue of determining the optimal electron beam radius for a given antiproton beam emittance and cooling section beta function. Suppose one decides to cool 95% of all antiprotons. Than the electron beam radius, r, should be:

$$r^2 = \varepsilon_{95\%}\beta \tag{3.3.3}$$

where $\varepsilon_{95\%}$ is the 95% emittance and β is the beta function of the beam in the cooling section. The electron beam cooling system is designed with the electron beam radius of about 6 mm and the beta-function of 30 m. This would allow cooling of beams with a transverse 95% emittance of about 1.5π-mm-mrad

The transverse velocity is proportional to the particle's angle

$$v_\perp \propto \sqrt{\frac{A}{\beta}} \tag{3.3.4}$$

where A is the betatron action of the particle. Since the electron beam has a uniform density, the longitudinal cooling rate is proportional to:

$$\lambda_\parallel \propto \frac{I}{\varepsilon_{95\%}} \frac{1}{Av_\parallel} \tag{3.3.5}$$

where I is the electron beam current. The transverse cooling rate is then proportional to:

$$\lambda_\perp \propto \frac{I}{\varepsilon_{95\%}} \sqrt{\frac{\beta}{A^3}} \tag{3.3.6}$$

Since the longitudinal cooling rate is independent of the beta function, large beta functions (which might have undesirable lattice properties) are not necessary.

The maximum electron cooling rate (for antiprotons with small amplitudes) is a function of the electron beam quality. A measure of the electron beam quality is the angular spread of the electron beam in the cooling section. The design for the rms angular spread specification in the electron beam is 0.1 mrad. This spread is determined by the following factors:

- Cooling section solenoid field quality
- Aberrations in the beamline
- Stability of the antiproton orbit
- Stability of the electron optics
- Emittance and space charge
- Stray magnetic fields.

The electron angular spread can be smaller than 0.1 mrad but it should not be greater than this value because this will reduce the cooling rates for large amplitude particles as compared to Equations (3.3.1) - (3.3.2), valid for a zero-temperature electron beam. In the other words, it is best if the electron rms angular spread is less or equal to the antiproton rms angular spread in the cooling section:

$$\sqrt{\frac{\varepsilon_{95\%}}{6\beta}} \approx 0.1\text{mrad}$$

(3.3.7)

This criterion will be met for a beta function of 30 meters when the emittance is about 1.5π-mm-mrad.

3.3.2 Recycler Stochastic Cooling

Because of the low beta function in the electron cooling section, the transverse electron cooling will not become effective until the antiproton beam emittance is below 1.0π-mm-mrad. To pre-cool the injected batch transverse emittance from 1.5π-mm-mrad to below 1.0π-mm-mrad, transverse stochastic cooling systems have to be used.

Because of the very low revolution frequency in the Recycler, the maximum frequency of the cooling stochastic systems is limited by bad mixing. To keep the phase error less than 45 degrees for the outside momentum particles:

$$f_{max} < \frac{f_o}{4x\eta \frac{\Delta pc}{pc}}$$

(3.3.8)

where x is the fraction of the circumference between pickup and kicker (x = 1/6 for the Recycler). Since the momentum spread is given as:

$$\Delta pc = \frac{\varepsilon_L}{T_{barrier}}$$

(3.3.9)

where ε_L is the longitudinal emittance of the beam and $T_{barrier}$ is the length of the beam pulse in between the barrier buckets, a maximum frequency of 4 GHz can handle 60 eV-Sec contained in a 1.6 uS long pulse. From Equation (3.2.10), the optimum cooling time for a stochastic cooling system is given as:

$$\frac{1}{\tau_{opt}} = \frac{W}{N}\frac{\Delta f_o}{f_o}\frac{f_c}{f_o}$$

(3.3.10)

where f_c is the center frequency of the cooling system. However, if the particles are not spread uniformly around the circumference, then the effective number of particles becomes:

$$N_{eff} = \frac{N}{f_o T_{barrier}}$$

(3.3.11)

The optimum cooling time becomes:

$$\tau_{opt} = \frac{1}{\eta}\frac{N}{f_c W}\frac{pc}{\varepsilon_L}$$

(3.3.12)

For a 2.5-3.5 GHz system, $22\text{x}10^{10}$ particles contained in 60 eV-Sec, the cooling time is 20 minutes.

3.3.3 Electron Cooling Rates

Every time an antiproton passes through the electron beam, it gets a tiny kick against its relative velocity. These kicks, averaged over the betatron phases, yield the electron cooling rates. Generally, the three electron cooling rates (x, y, and z) of the cooled particle are functions of all its three amplitudes. They are expressed in terms of multi-dimensional integrals over the electron velocity distribution, the cooler length and the particle betatron phases. For simulations, an analytical fit for the electron cooling rates has been used, where the electron angles were modeled as a transverse temperature described by a certain rms angle in the cooling section.[14] Formulas for this fit of electron cooling rates are expressed in terms of elementary and special (Bessel) functions. The fit inaccuracy is believed to be not worse than 20-30%. Plots illustrating some features of electron cooling rates, calculated for the cooler parameters of Table 3.3-1, are presented in Figure 3.3-1 and Figure 3.3-2. In both figures, the transverse action is the Courant-Snyder invariant defined so that its beam average gives the normalized rms emittance. The red trace corresponds to the second action of 0.5π mm mrad (normalized) and equivalent to that longitudinal velocity in the beam frame. The blue trace is for a 4 times larger second action and the same longitudinal velocity as the red trace. The brown trace relates to the same second action and 2 times larger longitudinal velocity compared with the red trace.

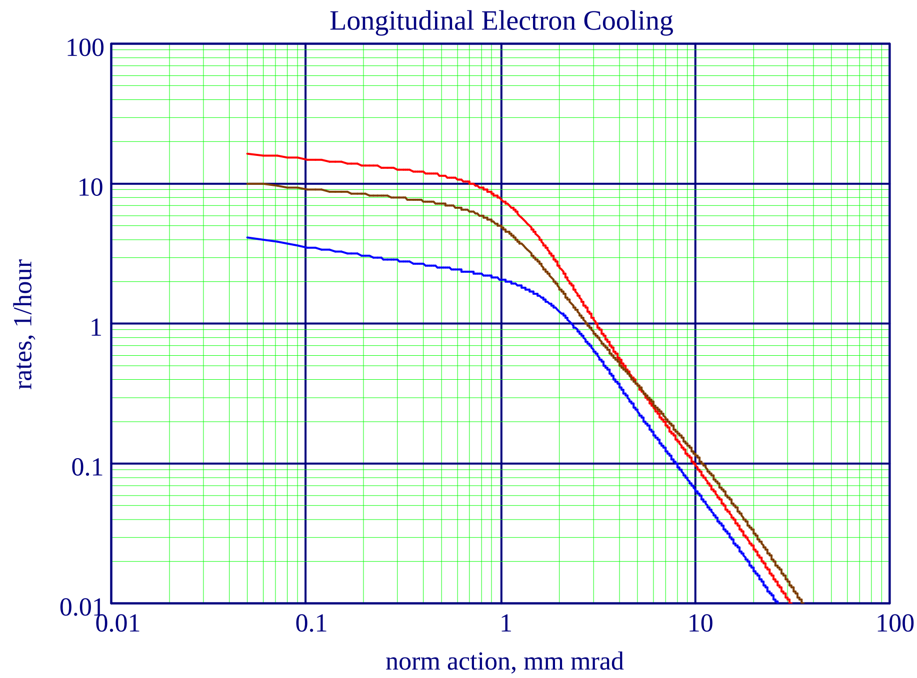

Figure 3.3-1 Longitudinal electron cooling rates as functions of one of the normalized transverse actions.

[14] Stacking in the Recycler, Run II Upgrades, A. Burov, V. Lebedev, 2003

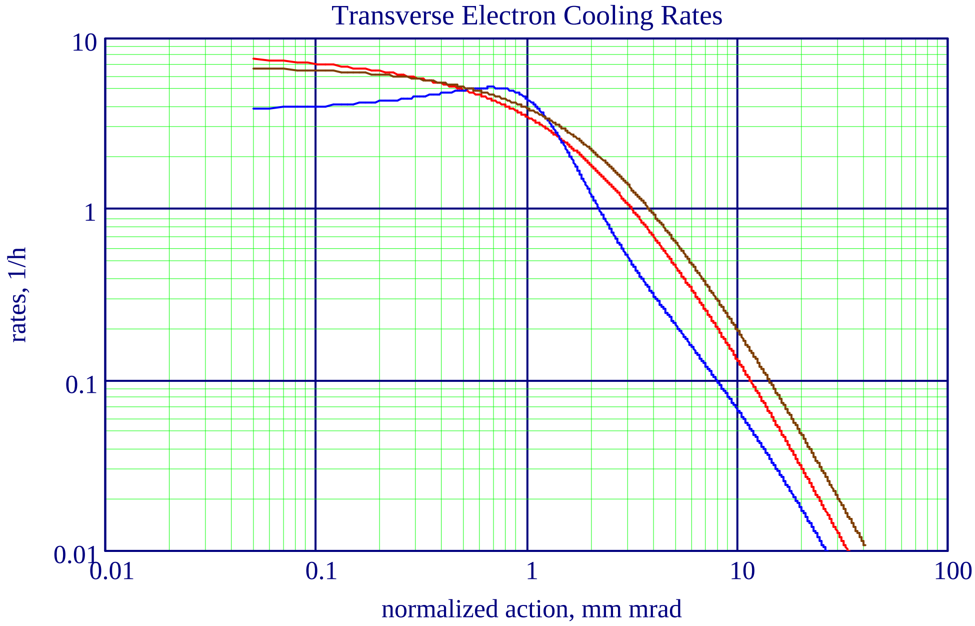

Figure 3.3-2 Transverse electron cooling rates.

3.3.4 Electron Cooling Simulations

The cooling-stacking process with transverse gated stochastic cooling and three-dimensional electron cooling of the stack was modeled with Monte-Carlo simulations.[14] The stochastic cooling with its cooling and diffusion terms renormalized by the feedback through the beam is taken into account in the conventional way. Electron cooling rates are functions of the three antiproton actions and take into account finite electron-beam radius and transverse temperature. The model calculates the evolution of the distribution for given values of input parameters such as initial emittances, transverse and longitudinal diffusion coefficients, injection rate, batch and stack intensities, bunching factors, and bandwidth of the stochastic cooling system as shown in Table 3.3-1. The simulation consists of two parts: the transverse stochastic pre-cooling of the batch during the repetition period, and the combined electron-stochastic cooling of this batch merged with the stack prior to the next repetition period. When the stack is merged with the batch, its longitudinal emittance increases. As a requirement for self-consistency, the stack emittances must be cooled to the same values at the end of each repetition period. Also, the number of particles in the batch is so small that intra-beam scattering is not significant. The bunching factor for the stack is determined by the requirement of thermal equilibrium of the longitudinal and transverse emittances; thus, it varies during the cooling process.

The evolution of the transverse distribution of the stack is shown in Figure 3.3-3. The distribution integral (fraction of particles outside a given action) is presented just after injection (red trace), after 30 minutes of the gated stochastic cooling (magenta trace), after 15 more minutes being merged and cooled with the stack (blue trace), and right before the next merge (black trace). The longitudinal evolution is shown in Figure 3.3-4. The distribution integral is shown for the 30 minutes of electron cooling. The red trace shows the state right after the merge, then cyan, magenta, blue and black traces depict the distribution after every 7.5 min.

Results of this particular simulation present several important features:

- The injection batch transverse emittance is stochastically cooled for 30 minutes from an initial value 15π-mm-mrad (95% normalized) to a final value 3 π-mm-mrad. This small final emittance is in approximate equilibrium between cooling and an external transverse diffusion of 8π-mm-mrad/hour.
- The total longitudinal phase space, which can be as high as 90 eV-sec (95%) just after the merger, is reduced to 30 eV-sec with 30 minutes of electron cooling.
- For this scenario, the stack bunching factor varies from 0.6 right after the merger to 0.2 just prior to the next merger. The electron current may be either DC or follow the same pattern.
- This simulation not the only solution for the required set of input parameters. For example, a higher value of the transverse diffusion would require more stochastic cooling and electron cooling.

Transverse stochastic cooling band	**2.5 – 3.5** GHz
Batch transverse emittances at injection, 95% norm	**10** π mm mrad
Batch longitudinal 95% phase area before pre-cooling	**60** eVs
Pbars per batch	$\mathbf{22.5 \cdot 10^{10}}$
Injection periodicity	**30** min
Pbars in the stack, up to	$\mathbf{600 \cdot 10^{10}}$
Stack transverse emittance before merger, 95% norm	**3** π mm mrad
Stack longitudinal 95% phase area before merger	**30** eVs
Peak electron current	**0.5** A
E-cooling length	**20** m
Electron 1D rms angle in the cooler	**0.22** mrad
Electron beam radius	**2.7** mm
Beta-function in the e-cooler	**22** m
Transverse diffusion (norm 95% emittance growth)	**8** π mm mrad / hour

Table 3.3-1 Parameters of the simulation shown in Figs. 8 and 9.

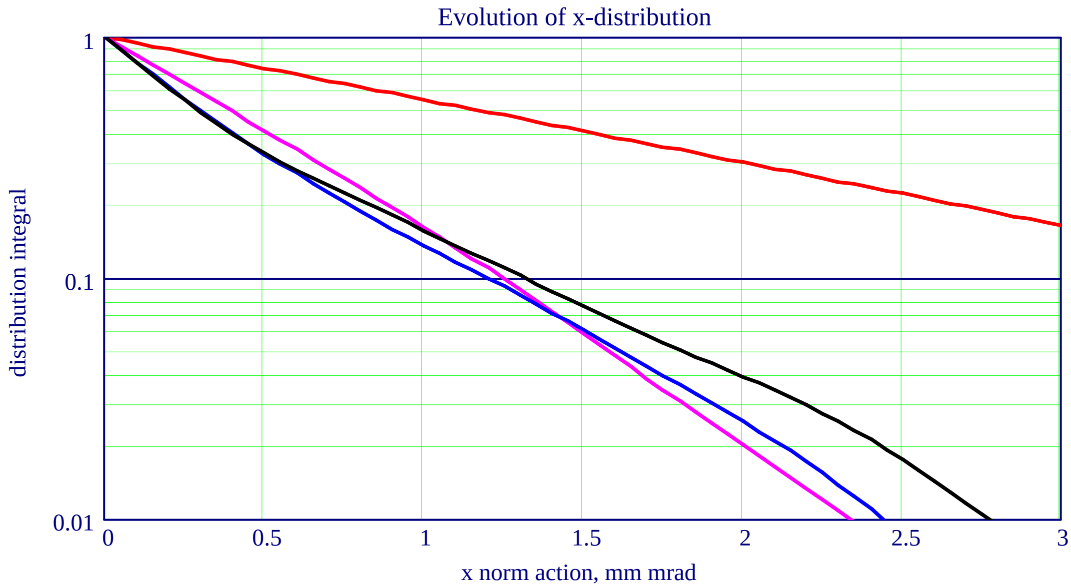

Figure 3.3-3 Evolution of the horizontal distribution of the stack.

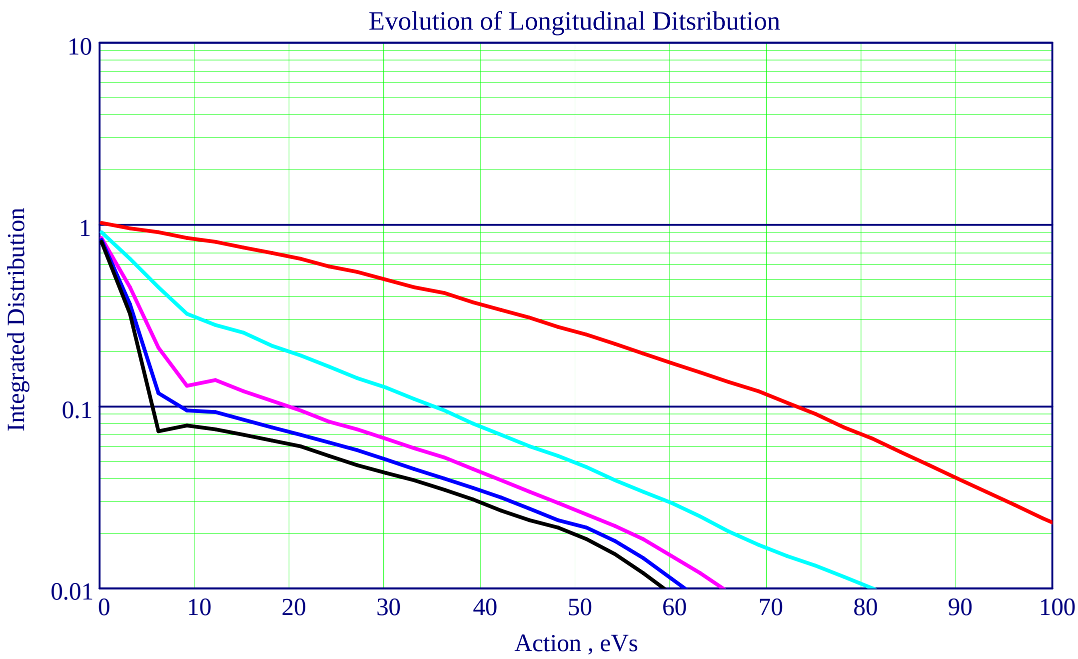

Figure 3.3-4 Longitudinal evolution

3.3.5 Coherent Instabilities

The space charge tune shift for the maximum number of particles in the cooled stack is calculated to be 0.08, which is not far from the limit of 0.10-0.15 observed in conventional electron coolers. This high tune shift suppresses Landau damping. Thus, the beam will probably be transversely unstable. To prevent instabilities, a broadband feedback system is required. The instability, driven by the resistive wall, is expected to be fastest at the low frequencies. The estimated growth time is about 300 turns. The upper frequency limit is determined by the Landau damping and is given as:

$$f \geq f_0 \frac{0.3\Delta\nu}{\eta(\Delta p / p)}.$$

This boundary could be as high as 0.7 GHz which is close to the lower frequency of the transverse stochastic cooling system.

4 Critical Parameters

Parameter	Value	Unit
Average Stacking rate	40	x10^{10} per hour
Peak Stacking rate	45	x10^{10} per hour
Number of particles injected into the Debuncher	280	x10^{6}
Debuncher transverse aperture	35	π-mm-mrad
Antiproton production cycle time	2	Secs
Maximum bunch length on target	1.5	nSecs.
Debuncher momentum aperture	4	%
Debuncher momentum cooling aperture	0.4	%
Debuncher final transverse emittance	5	π-mm-mrad
Debuncher final momentum spread	6	MeV
Debuncher transverse cooling common mode rejection	1.5	mm
Debuncher transverse cooling phase imbalance	3	degrees
Debuncher transverse cooling delay imbalance	1.4	pS
Debuncher momentum notch filter delay tolerance	1	pS
Debuncher momentum cooling notch filter dispersion	2.5	degrees
Debuncher to Accumulator transfer efficiency	95	%
Accumulator Stacktail Momentum bandwidth	2-6	GHz
Accumulator Core Momentum bandwidth	4-8	GHz
Accumulator Stacktail Momentum energy slope	8	MeV
Accumulator Stacktail Power	625	Watts
Accumulator Stacktail 2-6 GHz kicker impedance	6400	Ω
Accumulator Core Momentum energy slope	5	Mev
Accumulator Core Momentum cooling aperture	9.6	MeV
Accumulator Momentum cooling aperture	58	MeV
Accumulator to Recycler transfer longitudinal emmitance	10	eV-Sec
Accumulator to Recycler transfer interval	30	minutes
Number of particles extracted from the Accumulator per transfer	24	x10^{10}
Accumulator to Recycler transfer time	1	minutes
Accumulator to Recycler transfer efficiency	95	%
Accumulator core transverse emittance	1	π-mm-mrad

Table 3.3-1

Parameter	Value	Unit
Recycler transverse emittance injection dilution	50	%
Recyler longitudinal emmittance injection dilution	50	%
Peak Stack in Recycler	625	$x10^{10}$
Transverse emmitance of antiprotons extracted from Recycler	1	π-mm-mrad
Total Longitudinal emmitance of antiprotons extracted from Recycler	50	eV-Sec
Number of bunches extracted from the Recycler	36	
Minimum Electron Cooling Current	500	mA
Electron Beam alignment tolerance	0.22	mrad
Maximum transverse emittance for electron cooling	1.5	π-mm-mrad
Recycler Transverse stochastic cooling bandwidth	2.5 – 3.5	GHz
Recycler Injection Batch transverse emittances at injection	1.5	π mm mrad
Injection Batch longitudinal 95% phase area before pre-cooling	60	eV-sec
Electron cooling length	20	meters
Electron beam radius	2.7	mm
Beta-function in the e-cooler	22	meters
Transverse diffusion (norm 95% emittance growth)	8	π-mm-mrad/hour

Table 3.3-2

5 Study Plan

5.1 Debuncher Bunch Rotation

5.1.1 Experimentally verify the calculations of the final momentum spread as a function of proton bunch length (Figure 3.1-1)

5.2 Debuncher Transverse Stochastic Cooling

5.2.1 Document the cooling rate for a given starting emittance, power level, and number of particles for each band and all bands together.

5.2.2 Measure the signal to noise of each band for a given emittance and beam current and determine pickup impedance.[15,16]

5.2.3 Measure the common more rejection tolerances of each band

5.3 Debuncher Momentum Stochastic Cooling

5.3.1 Develop Fokker-Plank Computer simulations to account for dispersion properties of notch filters

5.3.2 Measure cooling rate and dispersion for each band and compare to simulations

5.3.3 Measure the signal to noise of each band for a given momentum spread and beam current

5.3.4 Measure the momentum aperture of the cooling system.

5.4 Accumulator Stacktail Momentum Stacking

5.4.1 Measure signal to noise and determine impedance of the Stacktail pickups, Core Momentum 2-4 GHz Pickups, and Core Momentum 4-8 GHz pickups

5.4.2 Characterize the beam transfer function as a function of energy for the Stacktail system, the Core 2-4 GHz

5.4.3 Develop detail Fokker-Plank model based on measurements in 5.4.1 and 5.4.2

5.4.4 Measure Stacktail pulse evolution as a function of initial distribution intensity, width, and position. Compare to model in 5.4.3.

---

[15] Pbar Note 564, Debuncher 4-8 GHz Pickup Tests, D. McGinnis, 1997
[16] Pbar Note 565, Debuncher 4-8 GHz Pickup Tests II, D. McGinnis, 1997

5.4.5 Measure zero stack Stacktail profile evolution as a function of initial distribution and pulse repetition rate. Compare to model in 5.4.3

5.5 Accumulator Transverse Cooling

5.5.1 Measure signal to noise and determine impedance of core transverse pickups.

5.5.2 Document beam transfer function measurements at the core.

5.5.3 Measure cooling rate as a function of stack size and system gain. Measure and subtract natural emittance growth of the accelerator from cooling measurements.

5.6 Recycler Electron Cooling

5.7 Recycler Stochastic Cooling

5.7.1 Measure signal to noise and determine impedance of transverse and longitudinal pickups.

5.7.2 Document beam transfer function measurements.

5.7.3 Measure cooling rate as a function of stack size and system gain. Measure and subtract natural emittance growth of the accelerator from cooling measurements.